necessary programming for carrying out this investigation. This work was performed under the auspices of the U.S. Atomic Energy Commission.

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A New Method Utilizing a Triangular Grid for the Representation of Preferred Orientation Using Re-Normalized *p* Values Obtained in Inverse Pole Figures

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A new method for plotting on one diagram the three inverse pole figures for each of the principal orthogonal sections in a flat rolled, extruded or drawn product has been tested. Such a diagram, designated the *texture triangle*, was found to offer the following advantages: (*a*) A clear idea of the orientation relationship for those planes which give rise to measurable X-ray reflexions can be obtained. (*b*) The change in texture with process variable may be plotted directly on the diagram. (*c*) Texture triangles may be plotted for a series of ideal textures and these may be used for comparative purposes. In this way the texture which is predominant in the specimen may be determined and the severity of the preferred orientation can be calculated in relation to the ideal texture.

Introduction

Before going into details of the texture triangle an outline of the inverse pole figure method will be given.

In the inverse pole figure technique (Harris, 1952) the diffraction intensities from a preferentially oriented sample are compared with those from a sample having an equal number of grains oriented in all directions. These relative intensities are called *texture coefficients* or *relative pole densities*, *p*, and are expressed mathematically (Morris, 1953; Mueller, 1954) as:

$$p = \frac{I_{hkl}/I^{\circ}_{hkl}}{\frac{1}{n}\sum_{0}^{n} I_{hkl}/I^{\circ}_{hkl}}, \qquad (1)$$

where

I = measured integrated intensity of a given hkl reflexion .

I° = calculated intensity for the same hkl reflexion as would be produced by a sample with equal number of grains oriented in all directions.

n = total number of reflexions measured.

p = pole density for a given reflexion.

The quantity I°_{hkl} in equation (1) is determined by the equation

$$I^{\circ}_{hkl} = (j)(\mathrm{Lp})(F^2)(e^{-2M}) \qquad (2)$$

where

j = multiplicity factor

Lp = Lorentz–polarization factor

F = structure factor
e^{-2M} = temperature factor .

The relative pole densities calculated from the above equation, are proportional to the number of grains in the given orientation. Hence

$p=1$ corresponds to a random orientation;
$p<1$ indicates a deficiency of planes parallel to the surface examined;
$p>1$ indicates an excess of planes parallel to the surface examined .

The relative pole densities calculated by this method can be plotted on a stereographic projection of the diffraction planes, which is called an inverse pole figure. Contour lines are drawn around the plotted points to include regions of similar p values in the same manner that iso-intensity lines are drawn on standard pole figures. The inverse pole figure gives a complete description of the grain orientation distribution for a single direction of the sample.

The inverse pole figure is very rapid and is generally best suited to the problems involved with fabricated shapes. The error in the p values is of the order of ±10%.

For sheet, three inverse pole figures can be made corresponding to the rolling, transverse and normal directions. For tubing, these inverse pole figures would be of the longitudinal, radial, and circumferential directions.

Twenty-four inverse pole figures are required if the process variables are altered eight times and it is, of course, necessary to look at all these inverse pole figures before formulating a picture of what happens to the preferred orientation when the process variables are altered.

This led the authors to bring about a new method of plotting the p values for any particular plane for the three orthogonal sections on a single triangular grid (Fig. 1).

Procedure

It was found necessary to renormalize the p values to corresponding $\bar{p}$ values before the values for the three orthogonal sections could be plotted on the same diagram. After altering the process variable, the new $\bar{p}$ values are plotted on the same diagram so that one can continually follow what happens to the orientation when the process variables are altered.

In the case of a rolled sheet the apices of the triangle could represent the rolling direction (R.D.), transverse direction (T.D.) and the normal direction (N.D.). The position of a plane, plotted in terms of its re-normalized p values for the three orthogonal directions in the triangle, indicates the tendency for the plane to lie perpendicular to either of the three directions. Thus if a point representing a particular plane lies close to the rolling direction (apex) this indicates that a large proportion of crystals are oriented with the same plane lying perpendicular to the rolling direction.

The procedure for re-normalizing is to adjust the p values so that the sum of the values in the three orthogonal sections studied for any crystallographic plane (hkl) is equivalent to the length of a triangle side which in this case is made to equal 90 divisions. This choice was made so that the distance between any two apices equals the angular relationship, in degrees, between the directions that these apices represent. 90 is therefore proportionally divided in the ratio of the three p values, and the results are the renormalized p values referred to as $\bar{p}$.

These values are then plotted by taking an intercept on the T.D.–R.D. side of the triangle for the $\bar{p}$ value of R.D., and an intercept on the N.D.–T.D. for the $\bar{p}$ value of the T.D. and an intercept on the R.D.–N.D. for the $\bar{p}$ value of N.D. The point where these three intercepts meet represents this particular orientation.

If a sample has a random orientation whereby the p value = 1 for any plane in any of the three orthogonal sections examined, then the $\bar{p}$ value = 30 and any plane would be represented by a point in the centre of the triangle. Any deviations from this would indicate the presence of a preferred orientation, the degree of which, would be represented by the distance from the centre of the triangular grid and its direction in space by the intercepts it makes on the three sides of the triangle.

Texture triangles may be plotted for a series of ideal textures and these may be used for comparative purposes. In addition the degree of preferred orientation can be calculated by comparing the distance between that point and the centre of the triangle to the distance between the position for the ideal texture and the centre of the triangle.

In cases where the ideal orientation is in the centre the degree of preferred orientation is best determined

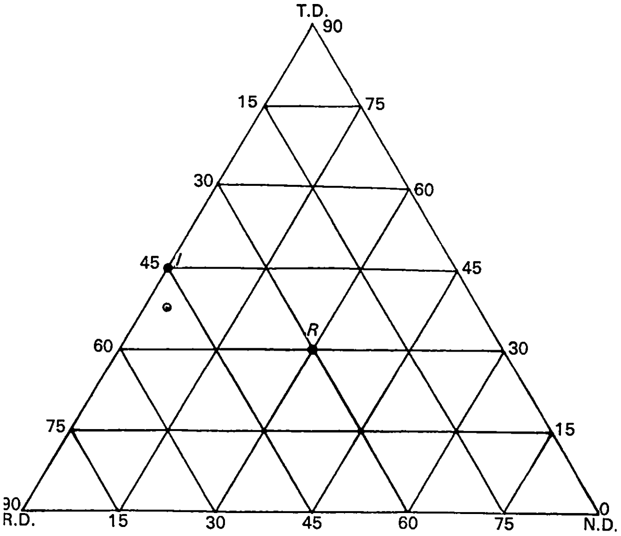


Fig. 1. Triangular grid with one point plotted on it at the $\bar{p}$ values of R.D. = 48·15, T.D. = 37·67, N.D. = 4·20.

by the relative pole densities p to prevent an ambiguous solution. The cases where the ideal texture is represented by a point at the centre of the triangle usually occur only in the cubic metals when the particular crystallographic plane in question, the (001) [100] in the cubic metals, reflects equally in all the three orthogonal directions. This ambiguity would not arise in the hexagonal and other crystal structures.

As an example of the method of plotting on the triangular grid an actual experimental example (Magee & Sanderson, 1967) will be utilized.

The cold deformation and recrystallization textures of unkilled mild steel, which initially was in the unannealed state having received 50% cold reduction, was examined.

For the 0% recrystallized sheet the p values for the (110) plane were as follows:

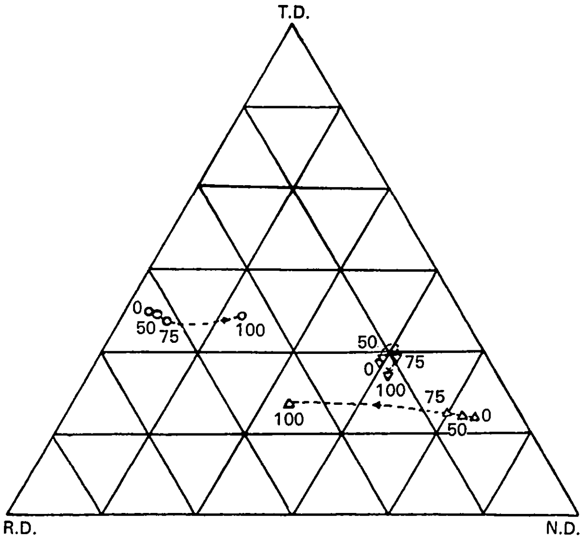
$$\begin{aligned} p \text{ value for the R.D.} &= 1\cdot590 \\ \text{T.D.} &= 1\cdot244 \\ \text{N.D.} &= 0\cdot137 \end{aligned}$$

$$p \text{ value for the three directions} = 2\cdot971\ .$$

These values are now re-normalized in the following manner so that the sum of the re-normalized p values = 90. The re-normalized p value ($\bar{p}$) for the R.D. is

$$\frac{1\cdot590}{2\cdot971} \times 90 = 48\cdot15$$

similarly the $\bar{p}$ value for the T.D. is 37·67 and for the N.D. is 4·20.



O = (110) △ = (200) ▽ = (211)

Fig. 2. Plot of 0, 50, 75, 100 % recrystallization $\bar{p}$ values for the three reflexions 110, 200, and 211 showing the distribution of the corresponding planes with respect to the three orthogonal directions. ○, 110; △, 200; ▽, 211.

These values are then plotted on the triangle (Fig. 1) as described previously. Here the preferred orientation is strong as compared with the position for a randomly oriented sample in the middle of the triangle (R). The (110) plane is nearly parallel to the N.D. (~5°) and lies between the T.D. and R.D. The severity of the texture can be represented by comparing the distance between this point and the centre of the triangle to that between the position for the ideal texture of this plane in the triangle (I) and the centre of the triangle.

In this case the texture, as a percentage of the ideal texture, ~90%.

When the above plot is applied to the 0, 50, 75 and 100% recrystallization, as determined by hardness measurements, it shows (Fig. 2) more clearly the distribution of each plane at each stage in the recrystallization process. The (112) planes are seen to be centred at a position R.D. $\simeq$ 15, T.D. $\simeq$ 30 and N.D. $\simeq$ 45 forming a circle in a clockwise motion during recrystallization. The (100) and (110) planes show little change up to 75% recrystallization but this is followed by a large change, both planes moving approximately to the centre of the triangle which represents the textures for a random sample as against the ideal texture.

Conclusion

As seen from the experimental results plotted with the triangular grid, this provides a method of plotting the orientation textures of three orthogonal sections on one diagram. New orientations obtained with varying the process variables can also be plotted on the same diagram, which permits direct observations and measurements. In addition the degree of the preferred orientation can be expressed as a percentage of the ideal textures which can also be plotted on the same diagram.

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